

Add larger numbers, using the formal written method of columnar addition.  
 Use their knowledge of the order of operations to carry out calculations involving the 4 operations.  
 Calculate intervals across zero.  
 Add fractions with different denominators and mixed numbers, using the concept of equivalent fractions.

### Add integers up to 10 million

Encourage children to estimate and use inverse operations to check answers to calculations.

	3	4	6	2	2	1
+	1	8	4	3	2	1
	1	1				
	5	3	0	5	4	2

?
2,354

	8	1		8	5
+			0	6	
	9	9	5		8

### Add decimals with up to 3 decimal places

Progress to numbers with digits in different place value columns.

Encourage children to check that they have lined up the columns correctly.

#### Order of operations

Calculations in brackets should be done first.  
 Multiplication and division should be performed before addition and subtraction.  
 \*When no brackets are shown and the operations have the same priority, work left to right.

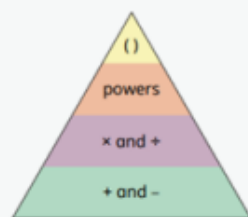
I do/do not need to make an exchange because ...

O	Tth	Hth	Thth
5	2	6	2

3	1	0	8
+	2	1	5
		1	
	5	2	6

1	5	0	2	7
+		9	5	8
	1	1		
	2	4	6	0

... has greater priority than ..., so the first part of the calculation I need to do is ...



$(3 + 4) \times 2 = 14$

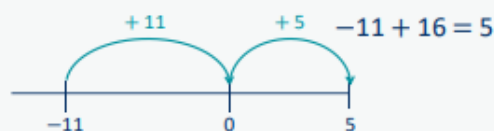
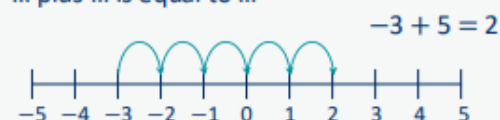
$3 + 4 \times 2 = 11$

$3 \times 4 + 2 = 14$

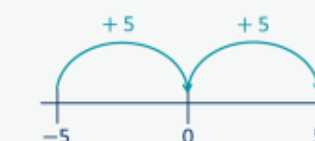
### Negative numbers

Children add to negative numbers and carry out calculations which cross 0

... plus ... is equal to ...



The difference between -5 and -1 is 4



The difference between -5 and 5 is 10

-1	1	+	1	6	=	5
	1	1		5		

## Order of operations

Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction.

\*When no brackets are shown and the operations have the same priority, work left to right.

... has greater priority than ..., so the first part of the calculation I need to do is ...



$$(3 + 4) \times 2 = 14$$



$$3 + 4 \times 2 = 11$$

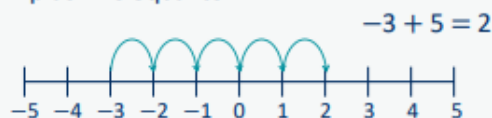


$$3 \times 4 + 2 = 14$$

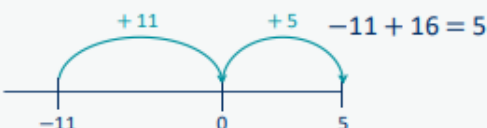
## Negative numbers

Children add to negative numbers and carry out calculations which cross 0

... plus ... is equal to ...



$$-3 + 5 = 2$$



$$-11 + 16 = 5$$



The difference between -5 and -1 is 4

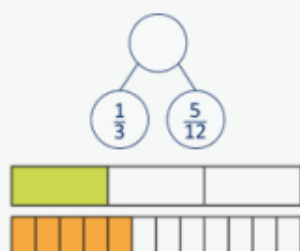


The difference between -5 and 5 is 10

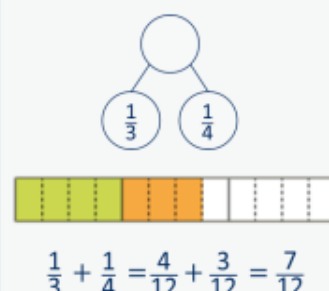
## Add fractions

Convert fractions to the same denominator before adding. Progress from fractions where one denominator is a multiple of the other, to any fractions and then to mixed numbers.

The denominator has been multiplied by ..., so the numerator needs to be multiplied by ...

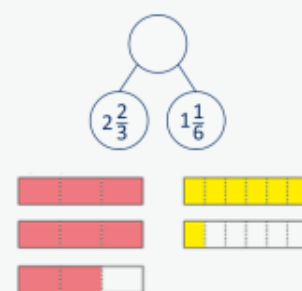


The lowest common multiple of ... and ... is ...



$$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

...is made up of ... wholes and ...



Subtract larger numbers, using the formal written methods of columnar subtraction.  
 Use their knowledge of the order of operations to carry out calculations involving the 4 operations.  
 Calculate intervals across zero.  
 Subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.

### Subtract integers up to 10 million

Encourage children to estimate and use inverse operations to check answers to calculations.

		2	1	5	1	2	1		
	-	1	8	4	3	2	1		
		1	6	1	9	0	0		

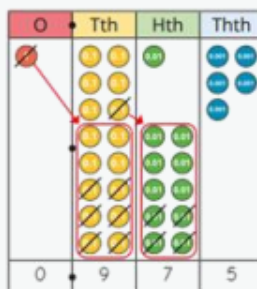
4,604		
2,354	750	?

		8		4	8	5	
	-	3	6				4
		5	5	5	5	5	

### Subtract decimals with up to 3 decimal places

Progress from the same number of decimal and whole number places to a different number of decimal and whole number places.

		6	7	1	3
	-	1	3	4	
		5	3	9	



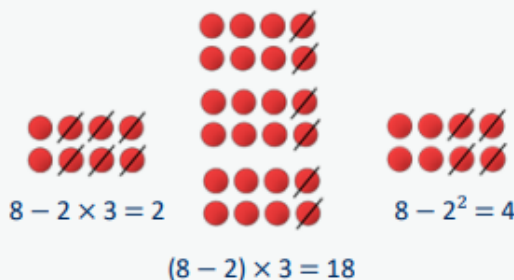
	1	6	1	5
-	0	6	4	0
	0	9	7	5

I do/do not need to make an exchange because ...

... has greater priority than ..., so the first part of the calculation I need to do is ...

### Order of operations

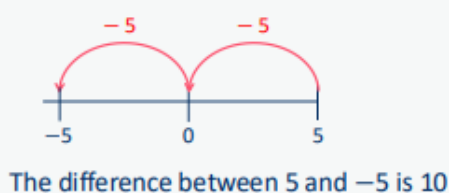
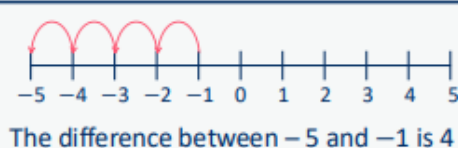
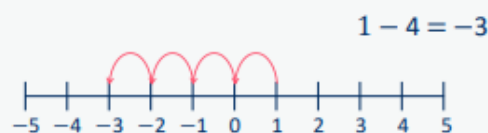
Children learn the order of priority for operations in a calculation. Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction.



### Negative numbers

Children subtract from positive and negative numbers and calculate intervals across 0

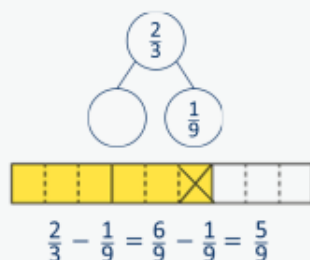
... minus ... is equal to ...



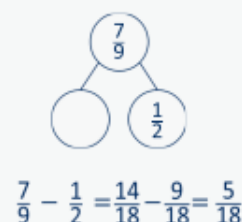
### Subtract fractions

Convert fractions to the same denominator before subtracting. Progress from fractions where one denominator is a multiple of the other, to any fractions and then subtracting from a mixed number.

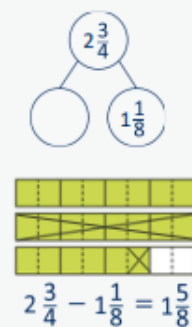
The denominator has been multiplied by ..., so the numerator needs to be multiplied by ...



The lowest common multiple of ... and ... is ...



... is made up of ... wholes and ...



Identify common factors and common multiples.

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.

### Multiply numbers by 10, 100 and 1,000

Multiply one-digit numbers with up to two decimal places by whole numbers.

Use their knowledge of the order of operations to carry out calculations involving the 4 operations.

Multiply simple pairs of proper fractions, writing the answer in its simplest form.  
Solve problems involving the relative sizes of two quantities where missing values  
can be found by using integer multiplication and division facts.

Solve problems involving the calculation of percentages.

## Representations

## Progression of Skills

### Multiply numbers up to 4 digits by a 2-digit number

To multiply by a 2-digit number, first multiply by the ones, then multiply by the tens and then find the total.

Handwritten multiplication of 1207 by 36 on grid paper. The calculation shows 1207 multiplied by 6 to get 7242, and 1207 multiplied by 30 to get 36210. These are then added to get the final result 43452. Blue arrows and lines highlight the carrying process and the final sum.

		1	2	0	7	
x				3	6	
		1				
		7	2	4	2	
+	3	6	2	1	0	
		4	3	4	5	2

(1,207 x 6)  
(1,207x30)

### Multiply by 10, 100 and 1,000

Some children may over-generalise that multiplying by a power of 10 always results in adding zeros.

To multiply by 10/100/1,000, I move all the digits ... places to the left.  
... is 10/100/1,000 times the size of ...

M	Hth	TTh	Th	H	T	O	Th	H	T	O	Tth	Hth	Thth
				●●	●●	●●				●●	●●	●●	●●

$$234 \times 10 = 2,340$$

$$234 \times 100 = 23,400$$

$$234 \times 1,000 = 234,000$$

$$0,234 \times 10 = 2,34$$

$$0,234 \times 100 = 23,4$$

$$0.234 \times 1,000 = 234$$

## Order of operations

Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction.

... has greater priority than ..., so the first part of the calculation I need to do is ...





$$(3 + 4) \times 2 = 14$$

3 + 4<sup>2</sup> = 19

3 + 4 × 2 = 11

### Multiply decimals by integers

This is the first time children multiply decimals by numbers other than 10, 100 or 1,000  
Encourage them to make links with known facts and whole number multiplication.

I know that  $\dots \times \dots = \dots$ ,  
so I also know that  $\dots \times \dots = \dots$

6 × 2 = 12      6 × 0.2 = 1.2

I need to exchange 10 ... for 1 ...

O	Tth	Hth


	3.	4	2
x			3
	1		
1	0.	2	6

$213 \times 4 = 852$        $2.13 \times 4 = 8.52$

### Multiply fractions by fractions

Encourage children to give answers in their simplest form.

When multiplying a pair of fractions, I need to multiply the numerator and multiply the denominator.



$$\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$$

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

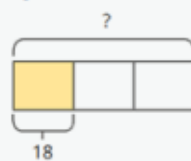
$\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$

## Find the whole

Children multiply to find the whole from a given part.

If  $\frac{1}{3}$  is ... , then the whole is ...  $\times$  ...

$$\frac{1}{3} \text{ of } \underline{\quad} = 18$$

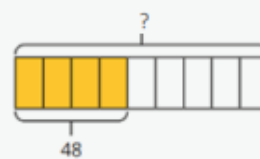


$$18 \times 3 = 54$$

$$\frac{1}{3} \text{ of } 54 = 18$$

If  $\frac{4}{9}$  is ... , then  $\frac{1}{9}$  is ... and the whole is ...  $\times$  ...

$$\frac{4}{9} \text{ of } \underline{\quad} = 48$$



$$\frac{1}{9} = 48 \div 4 = 12$$

$$9 \times 12 = 108$$

$$\frac{4}{9} \text{ of } 108 = 48$$

## Calculate percentages

Children first learn how to find 1%, 10%, 20%, 25% and 50% before using multiples of these amounts to find any percentage.

There are ... lots of ... % in 100%  
To find ... %, I need to divide by ...

100%			
50%		50%	
25%	25%	25%	25%

$$50\% \text{ of } \dots = \dots \div 2$$

$$25\% \text{ of } \dots = \dots \div 4$$

... % is made up of ... %, and ... %

100%									
10%	10%	10%	10%	10%	10%	10%	10%	10%	10%

To find 30%, I can find 10% and then multiply it by 3

To find 23%, I can use  $10\% \times 2$  and  $1\% \times 3$

To find 99%, I can find 1%, then subtract from 100%

## Calculations involving ratio

Encourage children to see the multiplicative relationship between ratios. They will need to multiply or divide each value by the same number to keep the ratio equivalent. Double number lines and ratio tables help children to see both horizontal and vertical multiplicative relationships.

For every ... , there are ...

For every 1 adult on a school trip, there are 6 children.

adults



children



Adults	Children
1	6
2	12
3	18

Diagram showing multiplicative relationships: A green arrow from 1 to 3 in the Adults column is labeled  $\times 3$ . A red arrow from 6 to 18 in the Children column is labeled  $\times 3$ . A green arrow from 6 to 12 in the Children column is labeled  $\times 2$ . A red arrow from 12 to 18 in the Children column is labeled  $\times 1.5$ .

The ratio of adults to children is 1 : 6





Perform mental calculations, including with mixed operations and large numbers.

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.

Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

Divide numbers by 10, 100 and 1,000 giving answers up to three decimal places.

Use written division methods in cases where the answer has up to two decimal places.

Associate a fraction with division and calculate decimal fraction equivalents.

Divide proper fractions by whole numbers [for example,  $1/3 \div 2 = 1/6$ ]

Solve problems involving the calculation of percentages.

### Short division

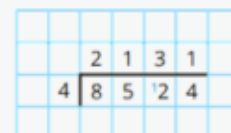
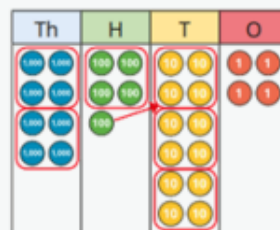
Encourage children to interpret remainders in context, for example knowing that "4 remainder 1" could mean 4 complete boxes with 1 left over so 5 boxes will be needed.

### Mental strategies

Include partitioning and number line strategies outlined in Y5 as well as division using factors.

There are ... groups of ... hundreds/tens/ones/ in ...

I can exchange 1 ... for 10 ...



To divide by ... , I can first divide by ... and then divide the answer by ...

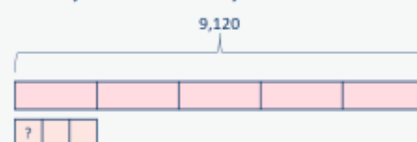
$$240 \div 60 = 240 \div 10 \div 6$$



$$480 \div 24 = 480 \div 4 \div 6$$

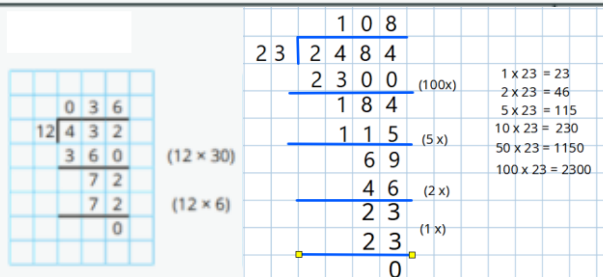


$$9,120 \div 15 = 9,120 \div 5 \div 3$$



### Long division

The long division method is introduced for the first time.



### Order of operations

Calculations in brackets should be done first, then powers. Multiplication and division should be performed before addition and subtraction.

... has greater priority than ..., so the first part of the calculation I need to do is ...



$$(6 + 4) \div 2 = 5$$

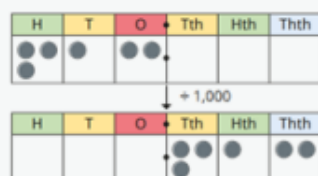


$$6 + 4 \div 2 = 8$$

### Divide by 10, 100 and 1,000

Encourage children to notice that dividing by 100 is the same as dividing by 10 twice, and that dividing by 1,000 is the same as dividing by 10 three times.

To divide by ... , I move the digits ... places to the right.



$$312 \div 10 = 31.2$$

$$312 \div 100 = 3.12$$

$$312 \div 1,000 = 0.312$$

$$906 \div 10 = 90.6$$

$$906 \div 100 = 9.06$$

$$906 \div 1,000 = 0.906$$

## Divide decimals by integers

This is the first time children divide decimals by numbers other than 10, 100 or 1,000

I know that  $\dots \div \dots = \dots$ ,  
so I also know that  $\dots \div \dots = \dots$



$$39 \div 3 = 13$$

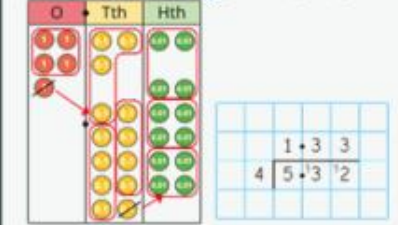


$$3.9 \div 3 = 1.3$$



$$0.39 \div 3 = 0.13$$

I need to exchange 1 ... for 10 ...



## Decimal and fraction equivalents

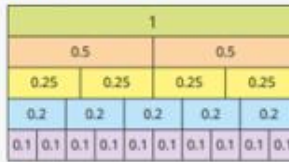
The fraction ... is equivalent to the decimal ...



$$\frac{1}{5} = 0.2$$

$$\frac{2}{5} = 0.4$$

$$\frac{3}{5} = 0.6$$



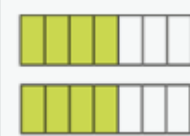
$\frac{\square}{\square}$  is equal to  $\frac{\square}{100}$

$$\frac{3}{4} = \frac{75}{100} = 0.75$$

## Divide a fraction by an integer

This is the first time children divide fractions by an integer.

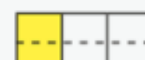
... ones divided by 2 is ... ones  
so ... sevenths divided by 2 is ... sevenths.



$$\frac{4}{7} \div 4 = \frac{1}{7}$$

$$\frac{4}{7} \div 2 = \frac{2}{7}$$

I am dividing by ..., so I can split each part into ... equal parts.



$$\frac{1}{3} \div 2 = \frac{1}{6}$$

... is equivalent to ...  
so  $\dots \div \dots = \dots \div \dots$

$$\frac{2}{3} = \frac{4}{6}$$

$$\text{so } \frac{2}{3} \div 4 = \frac{4}{6} \div 4 = \frac{1}{6}$$

## Fraction of an amount

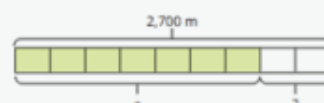
Children divide and multiply to find fractions of an amount. Bar models can still be used to support understanding where needed.

To find  $\frac{1}{\square}$  I divide by ...

$$\frac{1}{2} \text{ of } 36 = 36 \div 2$$

$$\frac{1}{12} \text{ of } 36 = 36 \div 12$$

If  $\frac{1}{\square}$  is equal to ..., then  $\frac{\square}{\square}$  are equal to ...



$$\frac{7}{9} \text{ of } 2,700 = \frac{1}{9} \text{ of } 2,700 \times 7$$

If  $\frac{\square}{\square}$  is equal to ..., then the whole is equal to ...

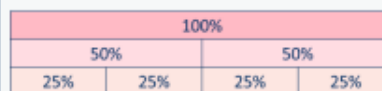


$$\frac{4}{9} \text{ of } \underline{\quad} = 48$$

## Calculate percentages

Children first learn how to find 1%, 10%, 20%, 25% and 50% before using multiples of these amounts to find any percentage.

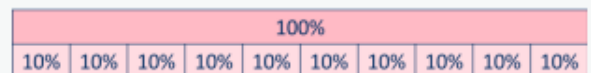
There are ... lots of ... % in 100%  
To find ... %, I need to divide by ...



$$50\% \text{ of } \dots = \dots \div 2$$

$$25\% \text{ of } \dots = \dots \div 4$$

... % is made up of ... %, and ... %



To find 30%, I can find 10% and then multiply it by 3

To find 23%, I can use  $10\% \times 2$  and  $1\% \times 3$

To find 99%, I can find 1%, then subtract from 100%

## Calculations involving ratio

Encourage children to see the multiplicative relationship between ratios. They will need to multiply or divide each value by the same number to keep the ratio equivalent. Double number lines and ratio tables help children to see both horizontal and vertical multiplicative relationships.

For every ..., there are ...

For every 6 children on a school trip, there is 1 adult.

adults



children



The ratio of children to adults is 6 : 1

